AJ**-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract:

In this chapter, a new class of sets αJ**-closed sets is initiated in topological spaces. Other than g-closed sets, analysis are performed on many stronger sets.

Key words: η*-closed, g-closed, J*-closed, J-closed, rg-closed.

1. Introduction: Regular open sets were first described in 1937 by Stone [1], and usedthem to define the semi-regularization of a topological spaces. Njastad [2] in 1968,introduced the concept of α-open sets which lies between open setsand semi-open sets. Generalized closed sets were introduced by Levine [3] in 1970. Dunham [4] has established a generalized closure using Levine's Generalized closed sets as Cl*. Annalakshmi [5] were introduced regular* open sets using Cl*. In 2018, Meenakshi. PL [6] introduced a class of new sets η*-open [6] which is placed between the classes of δ-open set and open set. In this paper, αJ**-closed sets are introduced using η*-open sets and their properties are explored.

2. Preliminaries:

Definition 2.1: Let (Y,ζ) be a topological space. If D is a non-empty subset of (Y,ζ) then the intersection of all closed sets containing D is called **closure of D** and is denoted by Cl(D). The union of all open sets contained in D is called **interiorof D** and is denoted by int(D).

Definition 2.2: Let A be a subset of a space (Y,ζ),

(i) The **generalized closure** of D [4] is defined as the intersection of all g-closed sets in Y containing D and is denoted by Cl*(D).

(ii) The **generalized interior** of D [4] is defined as the union of all g-open sets in Y containing D and is denoted by int*(D).

Definition 2.3: Let (Y,ζ) be a topological space. A subset D of space(Y,ζ) is called

- **Regular closed set** (Stone, 1937) if D=Cl(int(D))
- **Semi-closed set** (Levine, 1963) if int(Cl(D)) ⊆ D
- **α-closed set** (Njastad, 1965) if Cl(int(Cl(D))) ⊆ D
- **π-closed set** (Zaitsav, 1968) if it is the finit union of regular closed sets
- **pre-closed set** (Mashhour et al., 1982) if Cl(int(D)) ⊆ D
- **semi pre-closed set** (Andrijevic, 1986) if int(Cl(int(D))) ⊆ D

The complements of the above mentioned sets are called **regular open, semi-open,α-open, π-open and pre-open, semi pre-open sets** respectively.

The intersection of all **regular closed** (resp. **semi-closed,α-closed, π-closed, pre-closed and semi pre-closed**) subsets of (Y,ζ) containing D is called the **regular closure (**resp. **semi-closure, α-closure, π-closure, pre-closure and semi pre-closure)** of D and is denoted by rCl(D) (resp. sCl(D), αCl(D), πCl(D), pCl(D) and spCl(D)).

Definition 2.4: The **δ-interior** (Velicko, 1968) of a subset D of Y is the union of all regular open sets of Y contained in D and is denoted by $int_\delta(D)$. The subset D is called **δ-open** if D= $int_\delta(D)$, i.e. a set is δ-open if it is the union of regular open sets, the complement of δ-open is called **δ-closed.**Alternatively, a set D ⊆ Y is δ-closedif D = δCl(D), where **δCl(D)** is the intersection of all regular closed sets of (Y,ζ) containing D.

Definition 2.5: A subset D of a topological space (Y,ζ) is called **generalized -closed** (briefly **g-closed**)(Levine, 1970) if Cl(D) ⊆ M whenever D ⊆ M and Mis open in (Y,ζ). The complement of g-closed is a **g-open set.**

Definition 2.7: [Meenakshi PL, 2019]: A subset D of a topological space (Y,ζ) is called **η*-open set** if it is a union of regular*-open sets (r*-open sets). The complement of a η*-open set is called a **η*-closed set.** A subset D of a topological space (Y,ζ) is called **η*-Interior** of D is the union of a η*-open setsof Y contained in D. We denote the symbol by η*-Int(D). The intersection of all η*-closed sets of Y containing D is called **η*-closure** is denoted by η*-Cl(D).

Definition 2.8: A subset D of a topological space (Y,ζ) is called

1)**generalized semi-closed** (briefly **gs-closed**) (Arya et al., 1990) if sCl(D) ⊆ M whenever D ⊆ M and M is open in (Y,ζ).

2)regular generalized closed (briefly **rg-closed**) (Palaniappan, et. al.,1993) if Cl(D)⊆ M whenever D ⊆ M and M is regular open in (Y,ζ).

3) **regular weakly generalized closed** (briefly **rwg-closed**) (Nagaveni, 1999) if Cl(int(D)) ⊆ M whenever D ⊆ M and M is regular open in (Y,ζ).

4) **π-generalized closed** (briefly **πg-closed**) (Dontchev et.al.,2000)if Cl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y,ζ).

5) **generalized δ-closed** (briefly **gδ-closed**) (Dontchev, 2000) if sCl(D) ⊆ M whenever D ⊆ M and M is δ-open in (Y,ζ).

6) **π-generalized semi-closed** (briefly **πgs-closed**) (Aslim et.al.,2006)if sCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y,ζ).
7) **π-generalized pre-closed** (briefly **πgp-closed**) (Park, 2006)if pCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y,ζ).
8) **π-generalized α-closed** (briefly **πgα-closed**) (Janaki, 2009)if αCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y,ζ).
9) **π-generalized semi pre-closed** (briefly **πgsp-closed**) (Sarsak,2010)if spCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y,ζ).
10) **generalized semi pre regular-closed** (briefly **gspr-closed**) (Sarsak et.al.,2010) if spCl(D) ⊆ M whenever D ⊆ M and M is regular open in (Y,ζ).
11) **generalized pre regular-closed** (briefly **gpr-closed**) (Gnanambal, 1998) if pCl(D) ⊆ M whenever D ⊆ M and M is regular open in (Y,ζ).
12) **J-closed** [Meenakshi PL,2021] if Cl(D) ⊆ M whenever D ⊆ M and M is η*-open in (Y,ζ).
13) **semi generalized closed** (briefly **sg-closed**) (Bhattacharya et.al.,1987) if sCl(D) ⊆ M whenever D ⊆ M and M is semi-open in (Y,ζ).
14) **J*-closed** [Meenakshi PL,2021] if η*-Cl(D) ⊆ M whenever D ⊆ M and M is η*-open in (Y,ζ).
15) **α-generalized closed** (briefly **αg-closed**) (Maki et al., 1994) if αCl(D) ⊆ M whenever D ⊆ M and M is open in (Y,ζ).
16) **generalized semi-pre closed** (briefly **gsp-closed**) (Dontchev, 1995) if spCl(D) ⊆ M whenever D ⊆ M and M is open in (Y,ζ).
The complements of the above mentioned sets are called their respective open sets.

Remark 2.9 [6]:
(i) π-closed(open) → regular closed(open) → δ-closed(open) → η*-closed(open) → closed(open) → semi-closed(open) → semi pre-closed(open).
(ii) π-closed(open) → regular closed(open) → δ-closed(open) → η*-closed(open) → closed(open) → α-closed(open).
(iii) π-closed(open) → regular closed(open) → δ-closed(open) → η*-closed(open) → closed(open) → g-closed(open).
(iv) π-closed(open) → regular closed(open) → δ-closed(open) → η*-closed(open) → closed(open) → pre-closed(open).

Remark 2.10 [6]: For every subset D of Y,
(i) spCl(D) ⊆sCl(D) ⊆ Cl(D) ⊆ η*-Cl(D) ⊆δCl(D) ⊆rCl(D) ⊆ πCl(D).
(ii) αCl(D) ⊆ Cl(D) ⊆ η*-Cl(D) ⊆δCl(D) ⊆rCl(D) ⊆ πCl(D).
(iii) gCl(D) ⊆ Cl(D) ⊆ η*-Cl(D) ⊆δCl(D) ⊆rCl(D) ⊆ πCl(D).
(iv) pCl(D) ⊆ Cl(D) ⊆ η*-Cl(D) ⊆δCl(D) ⊆rCl(D) ⊆ πCl(D).

3. αJ-closed sets in Topological spaces:**

This section introduces the term αJ**-closed sets, which refers to a new class of generalized closed sets. An analysis is done for the relationship between αJ**-closed sets and different closed sets.

Definition3.1: A subset D of a topological space (Y,ζ) is said to be αJ**-closed η*-Cl(D)⊆M, M is α-open in (Y,ζ). The class of allαJ **-closed sets of (Y,ζ) is denoted by αJ**C(Y,ζ).

Proposition3.2: Every closed set is αJ**-closed but not conversely.

Proof: Let D be a closed set and M be any α-open set containing D. Since D is closed, then Cl(D)=D, η*-closed closed,
∴ Cl(D) ⊆ η*-Cl(D) ⊆ M, where D ⊆ M, η*-Cl(D)⊆M, D is αJ**-closed.

Counter example3.3: Let Y={p,q,r}, ζ={ φ,Y,{p} }, αO(Y, ζ)={ φ,Y,{p},{p,q},{p,r}} , η*C(Y, ζ)= {Y,{q,r} }.The subset {r} is αJ**-closed but not closed.

Proposition3.4: Every η*-closed set is αJ **-closed but not conversely.

Proof: Let D be a η*-closed set and M be any α-open set containing D. Since D is η*-closed, η*-Cl(D)=D. Therefore η*-Cl(D)=D⊆M. This implies η*-Cl(D) ⊆M. Hence D is αJ**-closed.

Counter example3.5: Let Y={p,q,r}, ζ={ φ,Y,{p,q} }, αO(Y, ζ)={ φ,Y,{p,q}} , η*C(Y, ζ)= {Y,φ}. The subset {r} is αJ**-closed but not η*-closed.

Proposition 3.6: Every δ-closed set is αJ **-closed but not conversely.

Proof: Let D be a δ-closed set and M be any α-open set containing D. Since D is δ-closed, δCl(D)=D. This implies δCl(D)=D⊆M, and since every δ-closed is η*-closed. η*-Cl(D) ⊆
δCl(D) ⊆M. Hence D isαJ**-closed.

Counter example 3.7: Let Y={p,q,r}, ζ={ φ,Y,{p,q} }, αO(Y, ζ)={ φ,Y,{p,q}} , η*C(Y, ζ)= {Y,φ}.The subset {q,r} is αJ**-closed but not δ-closed.

Proposition 3.8: Every αJ**-closed set is J*-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be open set containing D,
i) open set α-open set

ii) η^*-closed closed

$\therefore Cl(D) \subseteq \eta^*\text{-}Cl(D) \subseteq M$, which implies that D is J^*-closed set.

Counter example 3.9: Let $Y= \{p, q, r, s\}$, $\zeta =\{Y,\varphi,\{p\},\{p,q\}\}$, $\eta^*Cl(D)=\{Y, \varphi,\{r,s\},\{q,r,s\}\}$, $\alpha O(Y, \zeta)=\{ \varphi, Y, \{p,q\},\{p\},\{p,r\},\{p,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\}\}$. The subset $\{p,r\}$ is J^*-closed but not αJ^{**}-closed.

Proposition 3.10: Every αJ^{**}-closed set is J-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be a η^*-open set containing D. Since D is αJ^{**}-closed,

i) η^*open α-open set

ii) η^*-closed closed

$\therefore Cl(D) \subseteq \eta^*Cl(D) \subseteq M$

From the definition of αJ^{**}-closed set and J-closed set, we get the result,Every αJ^{**}-closed set is J-closed.

Counter example 3.11: Let $Y= \{p, q, r, s\}$, $\zeta =\{Y,\varphi,\{p\},\{p,q\}\}$, $\eta^*Cl(D)=\{Y, \varphi,\{r,s\},\{q,r,s\}\}$, $\alpha O(Y, \zeta)=\{ \varphi, Y, \{p,q\},\{p\},\{p,r\},\{p,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\}\}$. The subset $\{p,s\}$ is J-closed but not αJ^{**}-closed.

Proposition 3.12: Every αJ^{**}-closed set is g-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be any open set containing D,

i) open set α-open set

ii) η^*-closed closed

$\therefore Cl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ^{**}-closed set and g-closed set, we get the result,Every αJ^{**}-closed set is g-closed.

Counter example 3.13: Let $Y= \{p, q, r\}$, $\zeta=\{Y, \varphi,\{p\} \}$, $\alpha O(Y, \zeta)=\{ \varphi, Y,\{p\}, \{p,q\},\{p,r\}\}$, $\eta^*C(Y, \zeta)= \{Y, \varphi,\{q,r\}\}$.The subset $\{p,q\}$ is g-closed but not αJ^{**}-closed.

Proposition 3.14: Every αJ^{**}-closed set is gsp-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be any open set containing D,

i) Open set α-open set

ii) η^*-closed closed

$\therefore Cl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ^{**}-closed set and gsp-closed set, we get the result,Every αJ^{**}-closed set is gsp-closed.

Counter example 3.15: Let $Y= \{p, q, r\}$, $\zeta=\{Y, \varphi,\{p\},\{p,q\}\}$, $\alpha O(Y, \zeta)=\{ \varphi, Y,\{p\}, \{p,q\},\{q\}\}$, $\eta^*C(Y, \zeta)= \{Y, \varphi,\{r\},\{p,r\},\{q,r\}\}$.The subset $\{p,r\}$ is gsp-closed but not αJ^{**}-closed.

Proposition 3.16: Every αJ^{**}-closed set is πg -closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be π-open set containing D,

i) π-open set α-open set

ii) η^*-closed closed

$\therefore Cl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ^{**}-closed set and πg-closed set, we get the result,Every αJ^{**}-closed set is πg-closed.

Counter example 3.17: Let $Y=\{p,q,r,s\}$, $\zeta=\{Y,\varphi,\{p\},\{r\},\{p,q\},\{p,r\},\{p,q,r\},\{p,r,s\}\}$, $\eta^*Cl(D)=\{Y,\varphi,\{s\},\{r,s\},\{pq,s\}\}$, $\alpha O(Y, \zeta)=\{Y, \varphi ,\{p\},\{r\},\{p,q\},\{p,r\},\{p,q,r\},\{p,r,s\}\}$The subset $\{p,r,s\}$ is πg-closed but not αJ^{**}-closed.

Proposition 3.18: Every αJ^{**}-closed set is gpr-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be regular open set containing D,

i) regular open set α-open set

ii) η^*closed pre-closed

$\therefore pCl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ^{**}-closed set and gpr-closed set, we get the result,Every αJ^{**}-closed set is gpr-closed.

Counter example 3.19: Let $Y=\{p,q,r\}$, $\zeta=\{Y,\varphi,\{p\},\{q\},\{p,q\}\}$, $\eta^*Cl(D)=\{Y,\varphi,\{r\},\{p,r\},\{q,r\}\}$, $\alpha O(Y, \zeta)=\{Y, \varphi ,\{p\},\{q\},\{p,q\}\}$. The subset $\{p,q\}$ is gpr-closed but not αJ^{**}-closed.

Proposition 3.20: Every αJ^{**}-closed set is πgp-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be π-open set containing D,

i) π-open set α-open set

ii) η^*-closed pre-closed

$\therefore pCl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ^{**}-closed set and πgp-closed set, we get the result,Every αJ^{**}-closed set is πgp-closed.

Counter example 3.21: Let $Y=\{p,q,r,s\}$, $\eta^*Cl(D)=\{Y,\varphi,\{s\},\{r,s\},\{p,q,s\}\}$,$\zeta= \{Y, \varphi,\{p\},\{r\},\{p,q\},\{p,r\},\{p,q,r\},\{p,r,s\}\}$, $\alpha O(Y, \zeta)=\{Y, \varphi ,\{p\},\{r\},\{p,q\},\{p,r\},\{p,q,r\},\{p,r,s\}\}$

The subset $\{p,r,s\}$ is πgp-closed but not αJ^{**}-closed.

Proposition 3.22: Every αJ^{**}-closed set is πgsp-closed but not conversely.

Proof: Let D be a αJ^{**}-closed set and M be regular open set containing D,

i) regular open set α-open set

ii) η^*-closed semi pre-closed

$\therefore spCl(D)\subseteq\eta^*Cl(D)\subseteq M$

From the definition of αJ**-closed set and πgsp-closed set, we get the result, Every αJ**-closed set is πgsp-closed.

Counter example 3.23: Let Y={p,q,r,s},η*Cl(D)={Y,φ,{q,r,s},{r,s},{p,r,s}}, ζ={Y,φ,{p},{q},{p,q},{p,q,r},{p,q,s}}, αO(Y, ζ)={Y, φ ,{p},{q},{p,q},{p,q,r},{p,q,s}}

The subset {p,q,s} is πgsp -closed but not αJ**-closed.

Proposition 3.24: Every αJ**-closed set is rg-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be regular open set containing D,

i) regular open set α-open set

ii) η*closed closed

∴ Cl(D)⊆η*Cl(D)⊆ M

From the definition of αJ**-closed set and rg-closed set, we get the result,Every αJ**-closed set is rg-closed.

Counter example 3.25: Let Y={p,q,r}, ζ={Y,φ,{p},{p,q},{q}},
η*Cl(D)={Y,φ,{r},{p,r},{q,r}}, αO(Y, ζ)={Y, φ ,{p},{q},{p,q}}

The subset {p,q} is rg-closed but not αJ**-closed.

Proposition 3.26: Every αJ**-closed set is πgs -closed but not conversely.

Proof: Let D be a αJ**-closed set and M be π-open set containing D,

i) π-open set α-open set

ii) η* closed semi closed

∴ sCl(D) ⊆η*Cl(D)⊆ M

From the definition of αJ**-closed set and πgs- closed set, we get the result,Every αJ**-closed set is πgs-closed.

Counter example 3.27: Let Y={p,q,r,s}, η*Cl(D)={Y,φ,{r,s},{q,r,s},{p,r,s}}, ζ={Y,φ,{p},{q},{p,q},{p,q,r},{p,q,s}},αO(Y, ζ)={Y, φ ,{p},{q},{p,q},{p,q,r},{p,q,s}}

The subset {p,q,r} is πgs-closed but not αJ**-closed.

Proposition 3.28: Every αJ**-closed set is πgα-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be π-open set containing D,

i) π-open set α-open set

ii) η* closed semi closed

∴ sCl(D)⊆ η*Cl(D)⊆ M

From the definition of αJ**-closed set and πgα- closed set, we get the result,Every αJ**-closed set is πgα-closed.

Counter example 3.29: Let Y={p,q,r,s}, η*Cl(D)={Y,φ,{s},{r,s},{p,q,s}},
ζ= {Y, φ,{p},{r},{p,q},{p,r},{p,q,r},{p,r,s}},
αO(Y, ζ)={Y, φ ,{p},{r},{p,q},{p,r},{p,q,r},{p,r,s}}

The subset {p,r,s} is πgα-closed but not αJ**-closed.

The following diagram explain about αJ-closed set is weaker than other existing g-closed sets:**

Proposition 3.30: Every αJ**-closed set is gs-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be open set containing D,

i) open set α-open set

ii) η* closed semi closed

∴ sCl(D)⊆ η*Cl(D)⊆ M

From the definition of αJ**-closed set and gs- closed set, we get the result, Every αJ**-closed set is gs-closed.

Counter example 3.31: Let Y={p,q,r,s}, ζ ={Y,φ,{p},{p,q}}, η*Cl(D)={Y, φ,{r,s},{q,r,s}}, αO(Y, ζ)={ φ, Y, {p,q},{p},{p,r},{p,s},{p,q,r},{p,q,s},{p,r,s}}. The subset {p,s} is gs-closed but not αJ**-closed.

Proposition 3.32: Every αJ**-closed set is αg-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be open set containing D,

i) open set α-open set

ii) η* closed α-closed

∴ αCl(D)⊆ η*Cl(D)⊆ M

From the definition of αJ**-closed set and αg- closed set, we get the result,Every αJ**-closed set is αg-closed.

Counter example 3.33: Let Y={p,q,r}, ζ={Y,φ,{p},{p,q}}, η*Cl(D)={Y,φ,{q,r}}, αO(Y, ζ)={Y, φ ,{p},{p,r},{p,q}}.The subset {p,r} is αg-closed but not αJ**-closed.

Proposition 3.34: Every αJ**-closed set is rwg-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be regular open set containing D,

i) regular open set α-open set

∴ Cl(D)⊆ η*Cl(D)⊆ M

Cl (intD)⊆ η*Cl (int D)⊆ M

Cl (intD)⊆ M

From the definition of αJ**-closed set and rwg- closed set, we get the result, Every αJ**-closed set is rwg-closed.

Counter example 3.35: Let Y={p,q,r}, ζ={Y,φ,{p},{p,q},{q}}, η*Cl(D)={Y,φ,{r},{p,r},{q,r}}, αO(Y, ζ)={Y, φ ,{p},{q},{p,q}}

The subset {p,q} is rwg-closed but not αJ**-closed.

Proposition 3.36: Every αJ**-closed set is gspr-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be regular open set containing D,

i) regular open set α-open set

ii) η* closed semi pre-closed

∴ spCl(D)⊆ η*Cl(D)⊆ M

From the definition of αJ**-closed set and gspr- closed set, we get the result, Every αJ**-closed set is gspr-closed.

Counter example 3.37: Let Y={p,q,r,s}, η*Cl(D)={Y,φ,{r,s},{q,r,s},{p,r,s}}, ζ={Y,φ,{p},{q},{p,q},{p,q,r},{p,q,s}}, αO(Y, ζ)={Y, φ ,{p},{q},{p,q},{p,q,r},{p,q,s}}

The subset {p,q,s} is gspr-closed but not αJ**-closed.

Proposition 3.38: Every αJ**-closed set is gδ-closed but not conversely.

Proof: Let D be a αJ**-closed set and M be δ-open set containing D,

i) δ-open set α-open set

ii) η* closed closed

∴ Cl(D)⊆ η*Cl(D)⊆ M

From the definition of αJ**-closed set and gδ- closed set, we get the result, Every αJ**-closed set is gδ-closed.

Counter example 3.39: Y={p,q,r,s}, η*Cl(D)={Y,φ,{r,s},{q,r,s},{p,r,s}}, ζ={Y,φ,{p}, {q},{p,q}, {p,q,r},{p,q,s}}, αO(Y, ζ)={Y, φ ,{p},{q},{p,q},{p,q,r},{p,q,s}}

The subset {p,q,r} is gδ-closed but not αJ**-closed.

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