

δ J-CONTINUOUS FUNCTIONS

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Abstract:

Different kinds of δ J-continuous function are introduced.

Keywords: δ J- continuous, Strongly δ J- continuous, super continuous, totally-continuous .

Introduction

In 1968, Velico proposed δ -open sets which are stronger than open sets. Levine has brought generalized closed sets in 1970. Later in 2016, Meenakshi P L has introduced a new sets namely η^* -open sets, a union of r^* -open sets, which is placed between δ open set and open set. Then a new class of sets namely J-closed sets was introducing η^* -open sets in topological spaces. This class of J-closed sets is placed between that of generalized closed(g-closed) sets and generalized δ -closed sets. In the year 2020, Meenakshi P L has introduced the concept of J-continuous Functions. In 2022, Vethavarna K P has introduced δ J closed sets. In this chapter , δ J continuous functions, different kinds of δ J-continuous functions namely Quasi δ J –continuous functions, Totally δ J –continuous functions, Strongly δ J-continuous functions and Contra δ J –continuous functions are introduced. In section 1, a topological space is represented by (g, τ) .

1.Preliminaries

Definition 1.1: Let (g, τ) be a topological space. If D is a non-empty subset of (g, τ) then the intersection of all closed sets containing D is called closure of D and is denoted by $Cl(D)$.

The union of all open sets contained in D is called interior of D and is denoted by $int(D)$.

Definition 1.2: Let (g, τ) be a topological space. A subset D of space is called **Regular closed set** (Stone, 1937) if $D=Cl(int(D))$

Semi-closed set (Levine, 1963) if $int(Cl(D)) \subseteq D$

α -closed set (Njastad, 1965) if $Cl(int(Cl(D))) \subseteq D$

π -closed set (Zaitsav, 1968) if it is the finit union of regular closed sets

Pre-closed set (Mashhour et al., 1982) if $Cl(int(D)) \subseteq D$

Semi pre-closed set (Andrijevic, 1986) if $int(Cl(int(D))) \subseteq D$

The complements of the above mentioned sets are called **regular open, semi-open, α -open, π -open and pre-open, semi pre-open sets** respectively.

The intersection of all **regular closed** (resp. **semi-closed, α -closed, π -closed, pre-closed and semi pre-closed**) subsets of (g, τ) containing D is called the **regular closure** (resp. **Semi-closure, α -closure, π -closure, preclosure and semi pre-closure**) of D and is denoted by $rCl(D)$ (resp. $sCl(D)$, $\alpha Cl(D)$, $\pi Cl(D)$, $pCl(D)$ and $spCl(D)$).

Definition 1.3: The **δ -interior** (Velicko, 1968) of a subset D of Y is the union of all regular open sets of Y contained in D and is denoted by $int\delta(D)$. The subset D is called **δ -open** if $D= int\delta(D)$, i.e. a set is δ -open if it is the union of regular open sets, the complement of δ -open is called **δ -closed**.

Alternatively, a set $D \subseteq Y$ is δ -closed if $D = \delta Cl(D)$, where **$\delta Cl(D)$** is the intersection of all regular closed sets of (g, τ) containing D.

Definition 1.4: A subset D of a topological space (g, τ) is called **generalized Closed** (briefly **g-closed**) (Levine, 1970) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (g, τ) . The complement of g-closed is a **g-open set**.

Definition 1.5 [Pious Annalakshmi, 2016]: Let (Y, ζ) be a topological space. A subset D of (Y, ζ) is called **regular*-open (or r^* -open)** if $D=int(Cl^*(D))$. The complement of regular*-open set is called **regular*-closed set**. The union of all regular*-open sets of Y contained in D is called **regular*-interior** and is denoted by $r^*int(D)$. The intersection of all regular*-closed sets of Y containing D is called **regular*-closure** is denoted by $r^*Cl(D)$.

Definition 1.6 [Meenakshi PL, 2019): A subset D of a topological space (g, τ) is called **η^* -open set** if it is a union of regular*-open sets (r^* -open sets). The complement of a η^* -open set is called a **η^* -closed set**. A subset D of a topological space (g, τ) is called **η^* -Interior** of D is the union of a η^* -open sets of Y contained in D. We denote the symbol by $\eta^*-Int(D)$. The intersection of all η^* -closed sets of Y containing D is called **η^* -closure** is denoted by $\eta^*-Cl(D)$.

Definition 1.7: A subset D of a topological space (g, τ) is called

- 1) **generalized semi-closed** (briefly **gs-closed**) (Arya et al., 1990) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (g, τ) .
- 2) **regular generalized closed** (briefly **rg-closed**) (Palaniappan, et. al., 1993) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (g, τ) .
- 3) **regular weakly generalized closed** (briefly **rwg-closed**) (Nagaveni, 1999) if $Cl(int(D)) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (g, τ) .
- 4) **π -generalized closed** (briefly **πg -closed**) (Dontchev et.al., 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is π -open in (g, τ) .
- 5) **generalized δ -closed** (briefly **$g\delta$ -closed**) (Dontchev, 2000) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and M is δ -open in (g, τ) .
- 6) **π -generalized semi-closed** (briefly **πgs -closed**) (Aslim et.al., 2006) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and M is π -open in (g, τ) .
- 7) **π -generalized pre-closed** (briefly **πgp -closed**) (Park, 2006) if $pCl(D) \subseteq M$ whenever $D \subseteq M$ and M is π -open in (g, τ) .
- 8) **π -generalized α -closed** (briefly **$\pi g\alpha$ -closed**) (Janaki, 2009) if $\alpha Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is π -open in (g, τ) .
- 9) **π -generalized semi pre-closed** (briefly **πgsp -closed**) (Sarsak, 2010) if $spCl(D) \subseteq M$ whenever $D \subseteq M$ and M is π -open in (g, τ) .
- 10) **generalized semi pre regular-closed** (briefly **$gspr$ -closed**) (Sarsak et.al., 2010) if $spCl(D) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (g, τ) .
- 11) **generalized pre regular-closed** (briefly **gpr -closed**) (Gnanambal, 1998) if $pCl(D) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (g, τ) .
- 12) **J-closed** [Meenakshi PL, 2021] if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is η^* -open in (g, τ) .
- 13) **δ generalized -closed** (briefly **δg -closed**) (Dontchev, 1996) if $\delta Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (g, τ) .
- 14) **δ generalized star -closed** (briefly **δg^* -closed**) (Sudha, 2014) if $\delta Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is g -open in (g, τ) .
- 15) **g^* -closed** (Veerakumar, 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is g -open in (g, τ) .
- 16) **\hat{g} -closed** (Veerakumar, 2003) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is semi-open in (g, τ) .
- 17) **$\#gs$ -closed** (Veerakumar, 2005) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and M is $*g$ -open in (g, τ) .
- 18) **$*g$ -closed** (Veerakumar, 2006) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is \hat{g} -open in (g, τ) .
- 19) **g^*s -closed** (Pushpalatha et al., 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is gs -open in (g, τ) .
- 20) **δg_{\dagger}^* -closed** (Dontchev, 2000) if $\delta Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is δ -open in (g, τ) .
- 21) **J^* -closed** [Meenakshi PL, 2021] if $\eta^*Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is η^* -open in (g, τ) .
- 22) **J^{**} -closed** [Meenakshi PL, 2021] if $\eta^*Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is η^* -open in (g, τ) .

The complements of the above mentioned sets are called their respective open sets.

Remark 1.8 :

- (i) π -closed(open) regular closed(open) δ -closed(open) η^* -closed(open) closed(open) semi-closed(open) semi pre-closed(open).
- (ii) π -closed(open) regular closed(open) δ -closed(open) η^* -closed(open) closed(open) closed(open) α -closed(open).
- (iii) π -closed(open) regular closed(open) δ -closed(open) η^* -closed(open) closed(open) g -closed(open).
- (iv) π -closed(open) regular closed(open) δ -closed(open) η^* -closed(open) closed(open) pre-closed(open).

Definition 1.9 . A function $f: g \rightarrow h$ is said to be

- ❖ **J -Continuous** (Levine, 1970) if for every closed set U in (h, σ) , $f^{-1}(U)$ is a closed set in (g, τ) .
- ❖ **strongly J- continuous** (Levine, 1960) if the inverse image of every subset of (h, σ) is clopen in (g, τ) .

- ❖ δ –continuous (Noiri,1980) if for every δ -closed set U of (h,σ) , $f^{-1}(U)$ is a δ -closed set of (g,τ) .
- ❖ **Totally J- continuous** (Jain,1980) if the inverse image of every open set of (h,σ) is δ clopen in (g,τ)
- ❖ **Super J- continuous** (Munshi,1982) if for every closed set U of (h,σ) , $f^{-1}(U)$ is a δ closed set of (g,τ) .
- ❖ **g-continuous**(Balachandran et al.),if for every closed set U in (h,σ) , $f^{-1}(U)$ is a g closed set in (g,τ)
- ❖ **rg-continuous**(Palaniappan,et.al.),if $f^{-1}(U)$ is a rg closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **gs-continuous**(Devi et.al., 1993)if $f^{-1}(U)$ is a gs closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **Contra J-continuous**(Dontchev,1996)if the inverse image of every closed set of (h,σ) is δ open set in (g,τ)
- ❖ **δg -continuous**(Dontchev,1996)if $f^{-1}(U)$ is a δg closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **gpr-continuous**(Gnanambal,1997)if $f^{-1}(U)$ is a gpr closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **rwg-continuous**(Nagaveni , 1999) if $f^{-1}(U)$ is a rwg closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **$g\delta$ -continuous**(Dontchev,2000) if $f^{-1}(U)$ is a $g\delta$ closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **\hat{g} -continuous**(Veerakumar,2003)if $f^{-1}(U)$ is a \hat{g} open set in (g,τ) for every open set U in (h,σ)
- ❖ **πgp -continuous**(Park,2004) if $f^{-1}(U)$ is a πgp closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **πgs -continuous**(Aslim,2006)if $f^{-1}(U)$ is a πgs closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **$\pi g\alpha$ -continuous**(Park,2004) if $f^{-1}(U)$ is a $\pi g\alpha$ closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **πg -continuous**(Ekici et.al.,2007) if $f^{-1}(U)$ is a πg closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **πgp -continuous**(Park,2004) if $f^{-1}(U)$ is a πgp closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **πgsp -continuous**(Park,2004) if $f^{-1}(U)$ is a πgsp closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **gspr-continuous**(Devi et.al., 1993)if $f^{-1}(U)$ is a $gspr$ closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **g^*s -continuous**(Pushpalatha et.al.),if $f^{-1}(U)$ is a g^*s closed set in (g,τ) for every closed set U in (h,σ)
- ❖ **δg^* -continuous**(Pushpalatha et.al.),if $f^{-1}(U)$ is a δg^* closed set in (g,τ) for every closed set U in (h,σ)

2.2. δJ -continuous Function in Topological Spaces

2.2. δJ -continuous Function

The properties of δJ -continuous functions in topological spaces are introduced and some of their related properties are analysed here.

DEFINITION 2.2.1 : A function $f:(g,\tau) \rightarrow (h,\sigma)$ is said to be δJ –continuous function if the inverse image of every closed set in (h,σ) is δJ closed in (g,τ) .

Example 2.2.2 : Let $f: (g,\tau) \rightarrow (h,\sigma)$ be the many one into function defined by $f(a)=\{b\}, f(b)=\{c\}, f(c)=\{a\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g,\emptyset,\{a\}\}$ and $\sigma=\{h,\emptyset,\{a\},\{b,c\}\}$, $\tau^c=\{g,\emptyset,\{b,c\}\}$, $\sigma^c=\{h,\emptyset,\{b,c\},\{a\}\}$. Then f is δJ - Continuous as $\delta JC(g,\tau)=\{g,\emptyset,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$.

Proposition 2.2.3: A δJ -continuous function $f:(g,\tau) \rightarrow (h,\sigma)$ is a J- continuous function but the converse is not true.

Proof Given $f: (g,\tau) \rightarrow (h,\sigma)$ is a δJ –continuous function . Let U be any closed set in (h,σ) .Since f is a δJ continuous, f^{-1} inverse of (U) is δJ closed in (g,τ) .Then $f^{-1}(U)$ is J closed in (g,τ) .Hence f is J -continuous .

Counter Example 2.2.4: Let $f:(g,\tau) \rightarrow (h,\sigma)$ be the function defined by $f(a)=\{a\}, f(b)=\{c\}, f(c)=\{b\}$ Let $g=h=\{a,b,c\}$ with $\tau=\{g,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{h,\emptyset,\{a\},\{b,c\}\}$. Here $\tau^c=\{g,\emptyset,\{b,c\},\{a,c\},\{c\}\}$, $\sigma^c=\{h,\emptyset,\{a\},\{b,c\}\}$ and $\delta J(g,\tau)=\{g,\emptyset,\{c\},\{b,c\},\{a,c\}\}$.Then f is J - continuous as $JC=P(g)$ but not δJ continuous. Because for the closed sets $\{a\}$ in (h,σ) the inverse image are not δJ closed in (g,τ) .

Proposition 2.2.5: A δJ -continuous function $f: (g,\tau) \rightarrow (h,\sigma)$ is a gs -continuous function but the converse is not true.

Proof Given $f:(g,\tau) \rightarrow (h,\sigma)$ is a δJ -continuous function. Let U be any closed set in (h,σ) .Since f is a δJ continuous, $f^{-1}(U)$ is δJ closed in (g,τ) .Then $f^{-1}(U)$ is gs closed in (g,τ) .Hence f is gs - continuous .

CounterExample 2.2.6: let $f:(g,\tau) \rightarrow (h,\sigma)$ be the function defined by $f(a)=\{c\}, f(b)=\{a\}, f(c)=\{b\}$ then $f^{-1}(a)=\{b\}, f^{-1}(b)=\{c\}, f^{-1}(c)=\{a\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g,\emptyset,\{a\},\{b,c\}\}$ and $\sigma=\{h,\emptyset,\{a,b\}\}$. Here $\sigma^c=\{h,\emptyset,\{c\}\}$, $\delta C(h,\sigma)=\{h,\emptyset,\{a\},\{b,c\}\}$ and δJ closed $(g,\tau)=\{g,\emptyset,\{b\},\{c\},\{a,b\},\{a,c\}\}$. Then f is gs -continuous as $gs(g,\tau)=P(g)$ but not δJ -continuous. Because for the closed set $\{c\}$ in (h,σ) , $f^{-1}(c)=\{a\}$ is not a δJ closed in (g,τ) .

Proposition 2.2.7: A δJ - continuous function $f:(g,\tau) \rightarrow (h,\sigma)$ is a πg -continuous function but the converse is not true.

Proof Given $f:(g,\tau) \rightarrow (h,\sigma)$ is a δJ -continuous function. Let U be any closed set in (h,σ) .Since f is a δJ -continuous, $f^{-1}(U)$ is δJ closed in (g,τ) .Then $f^{-1}(U)$ is πg closed in (g,τ) .Hence f is πg – continuous .

CounterExample 2.2.8: Let $f:(g,\tau) \rightarrow (h,\sigma)$ be the function defined by $f(a)=\{b\}, f(b)=\{c\}, f(c)=\{a\}$ then $f^{-1}(a)=\{c\}, f^{-1}(b)=\{a\}, f^{-1}(c)=\{b\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and $\sigma=\{h,\emptyset,\{a,b\}\}$. Here $\tau^c=\{g,\emptyset,\{b,c\},\{a,c\},\{c\},\{b\}\}$, $\sigma^c=\{h,\emptyset,\{c\}\}$, $\delta C(h,\sigma)=\{h,\emptyset,\{b\},\{a,c\}\}$ and $\delta J(g,\tau)=\{g,\emptyset\}$. Then f is πg - continuous as $\pi g(g,\tau)=P(g)$ but not δJ - continuous. Because for the closed set $\{b\}$ in (h,σ) $f^{-1}(b)=\{a\}$ is not δJ closed in (g,τ) .

Proposition 3.2.9: A δJ -continuous function $f:(g, \tau) \rightarrow (h, \sigma)$ is a πg_s – continuous function but the converse is not true.

Proof Given $f:(g, \tau) \rightarrow (h, \sigma)$ is a δJ -continuous function. Let U be any closed set in (h, σ) . Since f is a δJ -continuous, $f^{-1}(U)$ is δJ closed in (g, τ) . Then $f^{-1}(U)$ is πg_s closed in (g, τ) . Hence f is πg_s -continuous.

Counter Example 2.2.10: Let $f:(g, \tau) \rightarrow (h, \sigma)$ be the function defined by $f(a)=\{a\}, f(b)=\{c\}, f(c)=\{b\}$ then $f^{-1}(a)=\{a\}, f^{-1}(b)=\{c\}, f^{-1}(c)=\{b\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma=\{h, \emptyset, \{a,b\}\}$. Then $\tau^c=\{g, \emptyset, \{b,c\}, \{a,c\}, \{c\}\}$, $\sigma^c=\{h, \emptyset, \{c\}\}$, $\delta \mathcal{C}(h, \sigma)=\{h, \emptyset, \{c\}, \{a,c\}, \{b,c\}\}$ and $\delta J(g, \tau)=\{g, \emptyset, \{c\}, \{a,c\}, \{b,c\}\}$. Then f is πg_s -continuous as $\pi g_s(g, \tau)=\{g, \emptyset, \{a\}, \{b\}, \{c\}, \{a,c\}, \{b,c\}\}$ but not δJ -continuous. Because for the closed set $\{c\}$ in (h, σ) $f^{-1}(c)=\{b\}$ is not δJ closed in (g, τ) .

Proposition 2.2.11: A δJ -continuous function $f:(g, \tau) \rightarrow (h, \sigma)$ is a $gspr$ -continuous function but the converse is not true.

Proof Given $f:(g, \tau) \rightarrow (h, \sigma)$ is a δJ -continuous function. Let U be any closed set in (h, σ) . Since f is a δJ -continuous, $f^{-1}(U)$ is δJ closed in (g, τ) . Then $f^{-1}(U)$ is $gspr$ closed in (g, τ) . Hence f is $gspr$ -continuous.

Counter Example 2.2.12: Let $f:(g, \tau) \rightarrow (h, \sigma)$ be the identity function defined by $f(a)=\{a\}, f(b)=\{b\}, f(c)=\{c\}$ then $f^{-1}(a)=\{a\}, f^{-1}(b)=\{b\}, f^{-1}(c)=\{c\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g, \emptyset, \{a\}, \{b,c\}\}$ and $\sigma=\{h, \emptyset, \{a,b\}\}$. Then $\tau^c=\{g, \emptyset, \{a\}, \{b,c\}\}$, $\sigma^c=\{h, \emptyset, \{c\}\}$, $\delta \mathcal{C}(h, \sigma)=\{h, \emptyset, \{a\}, \{b,c\}\}$ and $\delta J(g, \tau)=\{g, \emptyset, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$. Then f is $gspr$ -continuous as $gspr(g, \tau)=P(g)$ but not δJ -continuous. Because for the closed set $\{a\}$ in (h, σ) $f^{-1}(a)=\{a\}$ is not δJ closed in (g, τ) .

Proposition 2.2.13: A δJ -continuous function $f:(g, \tau) \rightarrow (h, \sigma)$ is a $g\delta$ -continuous function but the converse is not true.

Proof Given $f:(g, \tau) \rightarrow (h, \sigma)$ is a δJ -continuous function. Let U be any closed set in (h, σ) . Since f is a δJ -continuous, $f^{-1}(U)$ is δJ closed in (g, τ) . Then $f^{-1}(U)$ is $g\delta$ closed in (g, τ) . Hence f is $g\delta$ -continuous.

Counter Example 2.2.14: Let $f:(g, \tau) \rightarrow (h, \sigma)$ be the identity function defined by $f(a)=\{a\}, f(b)=\{b\}, f(c)=\{c\}$ then $f^{-1}(a)=\{a\}, f^{-1}(b)=\{b\}, f^{-1}(c)=\{c\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g, \emptyset, \{a\}, \{b\}, \{a,c\}, \{a,b\}\}$ and $\sigma=\{h, \emptyset, \{a,b\}\}$. Then $\tau^c=\{g, \emptyset, \{b,c\}, \{a,c\}, \{b\}, \{c\}\}$, $\sigma^c=\{h, \emptyset, \{c\}\}$, $\delta \mathcal{C}(h, \sigma)=\{h, \emptyset, \{b\}, \{a,c\}\}$ and $\delta J(g, \tau)=\{g, \emptyset\}$. Then f is $g\delta$ -continuous as $g\delta(g, \tau)=P(g)$ but not δJ -continuous. Because for the closed set $\{b\}$ in (h, σ) $f^{-1}(b)=\{b\}$ is not δJ closed in (g, τ) .

Theorem 2.2.15: A δJ - function $f:(g, \tau) \rightarrow (h, \sigma)$ is a

1. πg_a continuous function
2. gpr -continuous function
3. rwg -continuous function
4. $gspr$ -continuous function
5. πg -continuous function
6. πgp -continuous function
7. πgs -continuous function

Proof Obvious

Remark 2.2.16 The converse of the above theorem 2.2.14 is not true. It can be seen from the following counter example.

Counter Example 2.2.17 In the above counterexample 2.2.14 $\pi g_a(g, \tau)=gpr(g, \tau)=rwg(g, \tau)=gspr(g, \tau)=\pi g(g, \tau)=\pi gp(g, \tau)=\pi gs(g, \tau)=P(g)$. Then f is πg_a -continuous, gpr -continuous, $gspr$ -continuous, πg -continuous, πgp -continuous, πgs -continuous but not δJ -continuous. Because for the closed set $\{b\}$ in (h, σ) , $f^{-1}(b)=\{b\}$ is not δJ closed in (g, τ) .

Theorem 2.2.18 A function $f:(g, \tau) \rightarrow (h, \sigma)$ is a δJ -continuous if and only if the inverse image of every closed set in (h, σ) is δJ -open in (g, τ) .

Proof Necessity Let $f:(g, \tau) \rightarrow (h, \sigma)$ be δJ continuous and U be any open set in (h, σ) . Then $h-U$ is closed in (h, σ) . Since f is δJ continuous, $f^{-1}(h-U)=h-f^{-1}(U)$ is δJ closed in (g, τ) and hence $f^{-1}(U)$ is δJ open in (g, τ) .

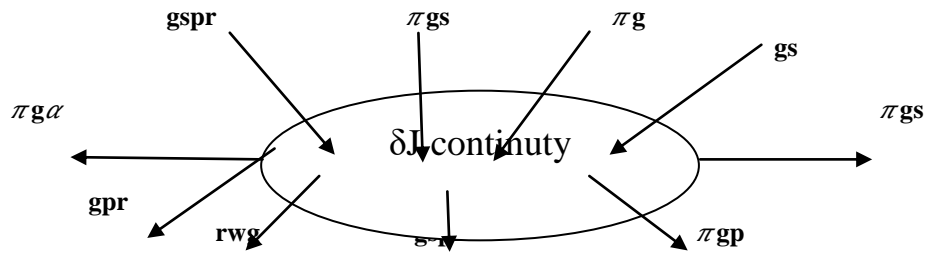
Sufficiency Assume that $f^{-1}(V)$ is δJ open in (g, τ) for each open set V in (h, σ) . Let V be a δ closed set in (h, σ) . Then $h-V$ is δ open in (h, σ) . By assumption $f^{-1}(h-V)=h-f^{-1}(V)$ is δJ open in (g, τ) which implies that $f^{-1}(V)$ is δJ closed in (g, τ) . Hence f is δJ -continuous.

Proposition 2.2.19 A super continuous function $f:(g, \tau) \rightarrow (h, \sigma)$ is a δJ - continuous function but the converse is not true.

Proof Given $f:(g, \tau) \rightarrow (h, \sigma)$ is a super continuous function. Let U be any closed set in (h, σ) . Since f is a super continuous function, $f^{-1}(U)$ is δ closed in (g, τ) . Then $f^{-1}(U)$ is a δJ closed in (g, τ) . Hence f is δJ -continuous.

Counter Example 2.2.20 Let $f:(g, \tau) \rightarrow (h, \sigma)$ be the function defined by $f(a)=\{b\}, f(b)=\{c\}, f(c)=\{b\}$ then $f^{-1}(c)=\{b\}, f^{-1}(b)=\{a,c\}$. Consider $g=h=\{a,b,c\}$ with $\tau=\{g, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma=\{h, \emptyset, \{a,b\}\}$. Here $\tau^c=\{g, \emptyset, \{c\}, \{b,c\}\}$, $\sigma^c=\{h, \emptyset, \{c\}\}$, $\delta \mathcal{C}(h, \sigma)=\{h, \emptyset, \{a\}\}$ and $\delta J(g, \tau)=\{g, \emptyset, \{b\}, \{c\}, \{a,b\}, \{a,c\}\}$. Then f is δJ continuous as $\delta \mathcal{C}(g, \tau)=\{g, \emptyset, \}$ but not super continuous. Because for the closed set $\{c\}$ in (h, σ) , $f^{-1}(c)=\{b\}$ is not δ closed in (g, τ) .

Remark 2.2.21 From the above discussions, we have the following diagram



Proposition 2.2.22 A totally continuous function $f:(g,\tau)\rightarrow(h,\sigma)$ is a δJ - continuous function but the converse is not true.

Proof Given $f:(g,\tau)\rightarrow(h,\sigma)$ is a totally continuous function. Let U be any open set in (h,σ) . Since f is a totally continuous, $f^{-1}(U)$ is clopen in (g,τ) . We know that “Every clopen is δJ open set”. Then $f^{-1}(U)$ is δJ open in (g,τ) . Hence f is δJ - continuous.

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