J*P-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract:
The aim of this paper is to introduce some new class of functions called J*P-continuous functions by using J*P-closed sets. The properties of J*P-continuous functions are discussed. Different kinds of continuous functions are introduced and characterized. Their interrelating properties are also obtained.
Keywords: J-continuous, gs-continuous, J*P-closed, g-continuous, strongly continuous.

Introduction:
Stone[19] (1937) defined the notion of regular open sets in his novel paper which related the theory of Boolean rings to General Topology. Mashhour et al[12] (1982) first studied the notion of pre open sets in topological spaces and obtained various properties. Levine[10] (1970) initiated the study of generalized closed sets in order to extend many of the important properties of closed sets to a larger family. Dunham (1982) has established a generalized closure using Levine's generalized closed sets as Cl*.In 2016, Annalakshmi has introduced regular*-open (r*-open sets) using CI*. Later in 2016, Meenakshi P L[14] has introduced a class of new sets namely η*- open sets, a union of r*-open sets, which is placed between δ-open set and open set. Meenakshi P L (2021) also introduced J*-closed sets and J**-closed sets. Its properties and characterization were established. Later, Malini R[13] (2022) has introduced J*P-closed sets and their properties are studied. In this paper, J*P-continuous functions are introduced.

1.Preliminaries:

Definition 1.1: Let (L,ρ) be a topological space. If D is a non-empty subset of (L,ρ) then the intersection of all closed sets containing D is called **closure of D** and is denoted by Cl(D).The union of all open sets contained in D is called **interior of D** and is denoted by int(D).

Definition 1.2: Let (L,ρ) be a topological space. A subset D of space (L,ρ) is called

- **regular closed set** (Stone, 1937)[19] if D=Cl(int(D))
- **semi-closed set** (Levine, 1963)[10] if int(Cl(D)) ⊆D
- **pre-closed set** (Mashhour et al., 1982)[12] if Cl(int(D)) ⊆D

The complements of the abovementioned sets are called **regular open, semi-open, and pre-open sets** respectively.

The intersection of all **regular closed** (resp. **semi-closed and pre-closed**) subsets of (L,ρ) containing D is called the **regular closure (**resp. **pre-closure and semi -closure)** of D and is denoted by rCl(D) (resp. sCl(D),αCl(D), πCl(D), pCl(D) and spCl(D)).

Definition 1.3: The **δ-interior** (Velicko, 1968) of a subset D of L is the union of all regular open sets of L contained in D and is denoted by $int_\delta(D)$. The subset D is called **δ-open** if D= $int_\delta(D)$, i.e., a set is δ-open if it is the union of regular open sets, the complement of δ-open is called **δ-closed.**

Alternatively, a set D ⊆Y is δ-closed if D = δCl(D), where **δCl(D)** is the intersection of all regular closed sets of (L,ρ) containing D.

Definition 1.4: A subset D of a topological space (L,ρ) is called **generalized Closed** (briefly **g-closed**) (Levine, 1970) if Cl(D) ⊆M whenever D ⊆M and M is open in (L,ρ). The complement of g-closed is a **g-open set.**

Definition 1.5: [Pious Annalakshmi, 2016]: Let (L,ρ) be a topological space. A subset D of (L,ρ) is called **regular*-open (or r*-open)** if D=int(Cl*(D)). The complement of regular*-open set is called **regular*-closed set.** The union of all regular*-open sets of L contained in D is called **regular*-interior** and is denoted by r*int(D).The intersection of all regular*-closed sets of L containing D is called **regular*-closure** is denoted by r*Cl(D).

Definition 1.6:[Meenakshi PL, 2019]: A subset D of a topological space (L,ρ) iscalled **η*-open set** if it is a union of regular*-open sets (r*-open sets). The complement of a η*-open set is called a **η*-closed set.** A subset D of a topological space (L,ρ) is called **η*-Interior** of D is the union of all η*-open sets of L contained in D. We denote the symbol by η*-Int(D). The intersection of all η*-closed sets of L containing D is called **η*-closure** and denoted by η*-Cl(D).

Definition 1.7: A subset D of a topological space (L,ρ) is called

1. **generalized semi-closed** (briefly **gs-closed**) (Arya et al., 1990) ifsCl(D) ⊆M whenever D ⊆M and M is open in (L,ρ).
2. **regular generalized closed** (briefly **rg-closed**) (Palaniappan, et.al.,1993) if Cl(D) ⊆M whenever D ⊆M and M is regular open in (L,ρ).
3. **regular weakly generalized closed** (briefly **rwg-closed**) (Nagaveni,1999) if Cl(int(D)) ⊆M whenever D ⊆M and M is regular open in (L,ρ).
4. **π-generalized closed** (briefly **πg-closed**) (Dontchev et.al.,2000)if Cl(D) ⊆M whenever D ⊆M and M is π-open in (L,ρ).

5. **generalized δ-closed** (briefly **gδ-closed**) (Dontchev, 2000) ifsCl(D) ⊆M whenever D ⊆M and M is δ-open in (L,ρ).
6. **π-generalized semi-closed** (briefly **πgs-closed**) (Aslim et.al.,2006)if sCl(D) ⊆M whenever D ⊆M and M is π-open in (L,ρ).
7. **π-generalized pre-closed** (briefly **πgp-closed**) (Park, 2006)if pCl(D) ⊆M whenever D ⊆M and M is π-open in (L,ρ).
8. **π-generalized α-closed** (briefly **πgα-closed**) (Janaki, 2009)if αCl(D) ⊆M whenever D ⊆M and M is π-open in (L,ρ).
9. **J*-closed** [Meenakshi PL,2021] if η*Cl(D) ⊆M whenever D ⊆M and M is η*-open in (L,ρ).

Remark 1.8: A topological space (L,ρ) is said to be a

1. **T_δ-space** (Dontchev, 2000) if every gδ-closed subset of (L,ρ) is δ-closed in (L,ρ).
2. **$_{J*P}T_\delta$-space** (Malini R,2022) when each J*P- closed set is δ-closed in (L,ρ).
3. **$_gT_{J*P}$-space** (Malini R,2022) when each g- closed set is J*P -closed in (L,ρ).
4. **$_{\alpha g}T_{J*P}$-space** (Malini R,2022) when each αg- closed set is J*P -closed in (L,ρ).
5. **$_{g\delta}T_{J*P}$-space** (Malini R,2022) when each gδ - closed set is J*P -closed in (L,ρ).
6. **$_{gs}T_{J*P}$-space** (Malini R,2022) when each gs- closed set is J*P -closed in (L,ρ).

Remark 1.9:

i. π-closed(open)→ regularclosed(open)→ δ-closed(open)→ η*-closed(open) → closed(open)→ semi-closed(open)→ semi preclosed(open).
ii. π-closed(open)→ regular closed(open)→ δ-closed(open)→ η*-closed(open)→ closed(open)→ closed(open)→ α-closed(open).
iii. π-closed(open)→ regular closed(open)→ δ-closed(open)→ η*-closed(open)→ closed(open)→ g-closed(open).
iv. π-closed(open)→ regular closed(open)→ δ-closed(open)→ η*-closed(open)→ closed(open)→ pre-closed(open).

Definition 1.10: A subset D is said to be J*P-closed if η*Cl(D)⊆M, whenever D⊆M and M is pre-open in (L,ρ).

Result 1.11: For a subset D of (L,ρ)

1.Cl(Y - D)= Y- int (D)

2.int(Y -D) =Y- Cl(D)

Definition 1.12: A function f:$(L,\rho) \to (T,\sigma)$is said to be

- ***s*trongly continuous (**Levine,1960) if the inverse image of every subset of (T,σ) is clopen in (L,ρ).
- **Continuous** (Levine,1970) if the inverse image of every closed set of (T,σ) is closed in (L,ρ).
- **δ-continuous (**Noiri,1980) if for every δ-closed set Vof (T,σ), $f^{-1}(V)$ is a δ-closed set of (L,ρ).
- **totally continuous** (Jain,1980) if the inverse image of every open set of (T,σ) is clopen in(L,ρ).
- **Super continuous (**Munshi,1982) if for every closed set Vof (T, σ), $f^{-1}(V)$ is a δ-closed set of (L,ρ).
- **g-continuous** (Balachandran et al.,)if for every closed set V in (T,σ), $f^{-1}(V)$ is a g-closed set in (L,ρ).
- ***r*g-continuous** (Palaniappan,et.al.,)if $f^{-1}(V)$ is a *r*g-closed set in (L,ρ) for every closed set V in (T, σ).
- **gs-continuous** (Devi et.al., 1993) if $f^{-1}(V)$ is a gs-closed set in (L,ρ) for every closed set V in (T, σ).
- **Contra continuous** (Dontchev,1996) if the inverse image of every closed set of (T,σ) is an open set in (L,ρ).
- **δg-continuous** (Dontchev,1996) if $f^{-1}(V)$ is a δg-closed set in (L,ρ) for every closed set V in (T, σ).
- **gpr-continuous** (Gnanambal,1997) if $f^{-1}(V)$ is a gpr-closed set in (L,ρ) for every closed set V in (T, σ).
- **rwg-continuous** (Nagaveni , 1999) if $f^{-1}(V)$ is a rwg-closed set in (L,ρ) for every closed set V in (T, σ).
- **gδ-continuous** (Dontchev,2000) if $f^{-1}(V)$is a gδ-closed set in (L,ρ) for every closed set V in (T, σ).
- **πgp-continuous** (Park,2004) if $f^{-1}(V)$ is a πgp-closed set in (L,ρ) for every closed set V in (T, σ).
- **πgs-continuous** (Aslim,2006) if $f^{-1}(V)$ is a πgs-closed set in (L,ρ) for every closed set V in (T, σ).
- **πgα-continuous** (Park,2004) if $f^{-1}(V)$ is a πgα-closed set in (L,ρ) for every closed set V in (T, σ).
- **πg-continuous** (Ekici et.al.,2007) if $f^{-1}(V)$ is a πg-closed set in (L,ρ) for every closed set V in (T, σ).
- **πgsp-continuous** (Park,2004) if $f^{-1}(V)$ is a πgsp-closed set in (L,ρ) for every closed set V in (T, σ).
- **gspr-continuous** (Devi et.al., 1993) if $f^{-1}(V)$ is a gspr-closed set in (L,ρ) for every closed set V in (T, σ).
- **g*s-continuous** (Pushpalatha et.al.,) if $f^{-1}(V)$ is a g*s-closed set in (L,ρ) for every closed set V in (T, σ).
- **δg*-continuous** (Pushpalatha et.al.,) if $f^{-1}(V)$ is a δg*-closed set in (L,ρ) for every closed set V in (T, σ).

2.J*P-Continuous Functions in Topological Spaces

2.2 J*P-Continuous Functions

The J*P-Continuous Functions in the topological spaces are introduced and investigated here.

Definition 2.2.1
A function f:$(L,\rho) \to (T,\sigma)$ is said to be J*P-continuous if the inverse image of every closed set in (T,σ) is J*P-closed in (L,ρ)

Example 2.2.2
Consider f:$(L,\rho) \to (T,\sigma)$ be the into function defined by f(a)=a, f(b)=b and f(c)=a. Let L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\}\}$, $\sigma = \{T,\emptyset,\{a\},\{a,b\}\}$ *and* $\sigma^c = \{T,\emptyset,\{c\},\{b,c\}\}$. Then f is J*P-continuous function as J*PC(L,ρ)={L,$\emptyset$,$\{b\},\{c\},\{b,c\},\{a,c\}$}.

Proposition 2.2.3
A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a J-continuous function but not conversely.

Proof:
Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ). Hence by proposition:2.2.5(Malini R [13]), $f^{-1}(V)$ is J-closed in (L,ρ). Hence f is J-continuous**.**

Counter Example 2.2.4
Let f:$(L,\rho) \to (T,\sigma)$ be the bijective function defined by f(a)=a, f(b)=c and f(c)=b. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\},\{a,b\},\{a,c\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b,c\}\}$. Here we have J*PC$(L,\rho)$={L,$\emptyset$,{b,c}} and σ^c={T, $\emptyset$, {a},{b,c}}. Then f is J-continuous as JC(L,ρ)=P(L) but not J*P-continuous function. Because for the closed set {a} in (T,σ), the inverse image is not J*P-closed in (L,ρ).

Proposition 2.2.5
A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a g-continuous function but not conversely.

Proof:
Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ). Hence by proposition:2.2.13 (Malini R [13]), $f^{-1}(V)$ is g-closed in (L,ρ). Hence f is g-continuous**.**

Counter Example 2.2.6
Let f:$(L,\rho) \to (T,\sigma)$ be the bijective function defined by f(a)=c, f(b)=b and f(c)=a. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\},\{b,c\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Here we have J*PC$(L,\rho)$={L,$\emptyset$,{a},{b,c}} and σ^c={T, $\emptyset$, {b}, {c},{a,c},{b,c}}. Then f is g-continuous as gC(L,ρ)=P(L) but not J*P-continuous function. Because for the closed set {b} in (T,σ), the inverse image is not J*P-closed in (L,ρ).

Proposition 2.2.7
A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a gs-continuous function but not conversely.

Proof:
Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ). Hence by proposition:2.2.19 (Malini R [13]), $f^{-1}(V)$ is gs-closed in (L,ρ). Hence f is gs-continuous**.**

Counter Example 2.2.8
Let f:$(L,\rho) \to (T,\sigma)$ be the into function defined by f(a)=a, f(b)=c and f(c)=c. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b,c\}\}$. Here we have J*PC$(L,\rho)$={L,$\emptyset$,{c},{b,c},{a,c}} and σ^c={T, $\emptyset$, {a},{b,c}}. Then f is gs-continuous as gsC(L,ρ)={L,$\emptyset$,{a},{b},{c},{a,c},{b,c}} but not J*P-continuous function. Because for the closed set {a} in (T,σ), the inverse image is not J*P-closed in (L,ρ).

Proposition 2.2.9
A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a rg-continuous function but not conversely.

Proof:
Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ). Hence by proposition:2.2.21 (Malini R [13]), $f^{-1}(V)$ is rg-closed in (L,ρ). Hence f is rg-continuous**.**

Counter Example 2.2.10
Let f:$(L,\rho) \to (T,\sigma)$ be the identity function defined by f(a)=a, f(b)=b and f(c)=c. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b,c\}\}$. Here we have J*PC$(L,\rho)$={L,$\emptyset$,{b,c}} and σ^c={T, $\emptyset$, {a},{b,c}}. Then f is rg-continuous as rgC(L,ρ)=P(L) but not J*P-continuous function. Because for the closed set {a} in (T,σ), the inverse image is not J*P-closed in (L,ρ).

Proposition 2.2.11
A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a gpr-continuous function but not conversely.

Proof:
Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ). Hence by proposition:2.2.25 (Malini R [13]), $f^{-1}(V)$ is gpr-closed in (L,ρ). Hence f is gpr-continuous**.**

Counter Example 2.2.12

Let f:$(L,\rho) \to (T,\sigma)$ be the into function defined by f(a)=a, f(b)=b and f(c)=b. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\},\{a,b\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Here we have J*PC$(L,\rho)$={L,∅,{b,c}} and σ^c={T, ∅, {b}, {c},{a,c}, {b,c}}. Then f is gpr-continuous as gprC(L,ρ)=P(L) but not J*P-continuous function. Because for the closed set {a,c} in (T,σ), f^{-1}({a,c})={a} is not J*P-closed in (L,ρ).

Proposition 2.2.13

A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a πg-continuous function but not conversely.

Proof:

Given f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function. Let V be any closed set in (T,σ). Since f is J*P-continuous function, f^{-1}(V) is J*P-closed in (L,ρ).Hence by proposition:2.2.29 (Malini R [13]), f^{-1}(V) is πg-closed in (L,ρ). Hence f is πg-continuous.

Counter Example 2.2.14

Let f:$(L,\rho) \to (T,\sigma)$ be the identity function. consider L=T={a,b,c} with $\rho = \{L,\emptyset,\{a\}\}$ and $\sigma = \{T,\emptyset,\{a\},\{b\},\{a,b\}\}$. Here we have J*PC$(L,\rho)$={L,∅,{b,c}} and σ^c={T, ∅, {c},{a,c},{b,c}}. Then f is πg-continuous as πgC(L,ρ)=P(L) but not J*P-continuous function. Because for the closed sets {c} and {a,c} in (T,σ), the inverse images are not J*P-closed in (L,ρ).

Theorem 2.2.15

A J*P-continuous function f:$(L,\rho) \to (T,\sigma)$ is a

i. rwg-continuous function
ii. πgp-continuous function
iii. πgs-continuous function
iv. πga-continuous function
v. gspr-continuous function
vi. πgsp-continuous function.

The proof is obvious.

Remark 2.2.16

The converse of the above theorem is not true.

Counter Example 2.2.17

Consider the **Counter Example 2.2.14,** we have rwgC(L,ρ)= πgpC(L,ρ)= πgsC(L,ρ)= πgaC(L,ρ)= gsprC(L,ρ)= πgspC(L,ρ)=P(L). Then f is rwg-continuous function, πgp-continuous function, πgs-continuous function, πga-continuous function, gspr-continuous function and πgsp-continuous function but not J*P-continuous function respectively. Because for the closed sets {c} and {a,c} in (T,σ), the inverse images are not J*P-closed in (L,ρ).

Theorem 2.2.18

A function f:$(L,\rho) \to (T,\sigma)$ is J*P-continuous if and only if the inverse image of every open set in (T,σ) is J*P-open in (L,ρ).

Proof:

Necessity:

Let f:$(L,\rho) \to (T,\sigma)$ be the J*P-continuous function and V be a open set in (T,σ). Then T-V is closed set in (T,σ). Since f is J*P-continuous, f^{-1}(T-V) =T-f^{-1}(V) is a J*P-closed in (L,ρ). Hence f^{-1}(V) is J*P-open in (L,ρ).

Sufficiency:

Assume that f^{-1}(U) is J*P-open in (L,ρ) for the each open set U in (T,σ). Let U be the closed set in (T,σ). Then T-U is the open set in (T,σ). By our assumption, f^{-1}(T-U) =T-f^{-1}(U) is J*P-open in (L,ρ) which implies that f^{-1}(U) is J*P-closed in (L,ρ). Hence f is J*P-continuous.

Proposition 2.2.19

A super continuous function f:$(L,\rho) \to (T,\sigma)$ is a J*P-continuous function but not conversely.

Proof:

Given f:$(L,\rho) \to (T,\sigma)$ is a super continuous function. Let V be any closed set in (T,σ). Since f is super continuous function, f^{-1}(V) is δ-closed in (L,ρ). Hence by proposition:2.2.11 (Malini R [13]), f^{-1}(V) is J*P-closed in (L,ρ). Hence f is J*P-continuous.

Remark 2.2.20

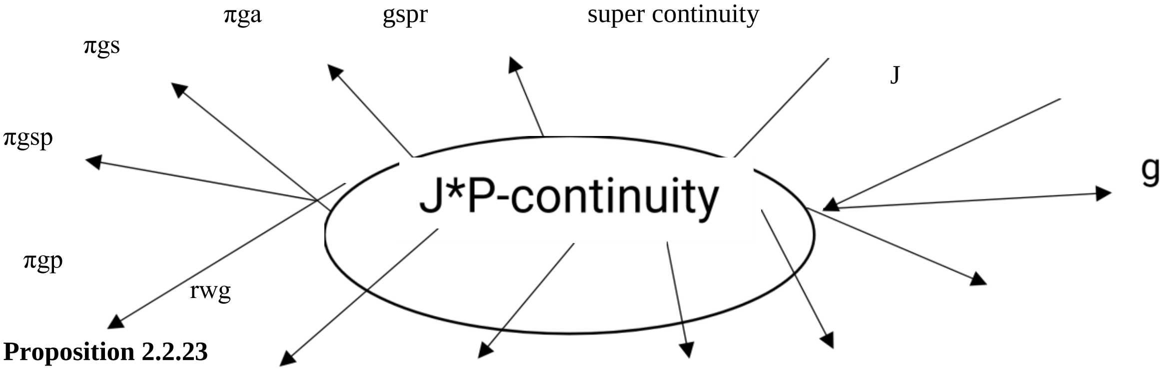
The converse of the above proposition is not true. Hence it can be seen from the following Counter Example.

Counter Example 2.2.21

Let f:$(L,\rho) \to (T,\sigma)$ be the into function defined by f(a)=a, f(b)=b and f(c)=b. consider L=T={a,b,c} with ρ={L,∅,{a},{a,b}} and $\sigma = \{T,\emptyset,\{a\}\}$. We have σ^c={T, ∅,{b,c}} and δC(L,ρ)={L,∅}. Then f is J*P-continuous function as J*PC(L,ρ)={L,∅,{b,c}} but not super continuous function. Because for the closed set {b,c} in (T,σ), the inverse image is not a δ-closed in (L,ρ).

Remark 2.2.22

From the above discussion, we have the following diagram:



Proposition 2.2.23
If f:$(L,\rho) \rightarrow (T,\sigma)$ is a gδ-continuous function and (L,ρ) is a $_{g\delta}T_{J*P}$-space. Then f is J*P-continuous.
Proof:
Given f:$(L,\rho) \rightarrow (T,\sigma)$ is a gδ-continuous function**.** Let V be any closed set in (T,σ). Since f is gδ-continuous function, $f^{-1}(V)$ is gδ-closed in (L,ρ) and (L,ρ) is a $_{g\delta}T_{J*P}$-space. Therefore $f^{-1}(V)$ is J*P-closed in (L,ρ)**.** Hence f is J*P-continuous.
Result 2.2.24

i. If f:$(L,\rho) \rightarrow (T,\sigma)$ is a gs-continuous function and (L,ρ) is a $_{gs}T_{J*P}$-space. Then f is J*P-continuous.
ii. If f:$(L,\rho) \rightarrow (T,\sigma)$ is a αg-continuous function and (L,ρ) is a $_{\alpha g}T_{J*P}$-space. Then f is J*P-continuous.
iii. If f:$(L,\rho) \rightarrow (T,\sigma)$ is a g-continuous function and (L,ρ) is a $_{g}T_{J*P}$-space. Then f is J*P-continuous.

Proof:
The proof is same as the **proposition 2.2.23**.
Proposition 2.2.25
If f:$(L,\rho) \rightarrow (T,\sigma)$ is a J*P-continuous function and (L,ρ) is a $_{J*P}T_{\delta}$-space. Then f is super continuous.
Proof:
Given f:$(L,\rho) \rightarrow (T,\sigma)$ is a J*P-continuous function**.** Let V be any closed set in (T,σ). Since f is J*P-continuous function, $f^{-1}(V)$ is J*P-closed in (L,ρ) and (L,ρ) is a $_{J*P}T_{\delta}$-space. Therefore $f^{-1}(V)$ is δ-closed in (L,ρ)**.** Hence f is super continuous.
Theorem 2.2.26
For every subset D of (L,ρ), f(J*Pcl(D))⊆cl(f(D)) if f:$(L,\rho) \rightarrow (T,\sigma)$ is a J*P-continuous function.
Proof:
Given f:$(L,\rho) \rightarrow (T,\sigma)$ is a J*P-continuous function and D is any subset of (L,ρ). Then cl(f(D)) is a closed set in (T,σ). Since f is J*P-continuous function, we get f^{-1}(cl(f(D))) is a J*P-closed set in (L,ρ) ----------(A). we know f(D)⊆cl(f(D)) which implies that D⊆f^{-1}(cl(f(D))). From (A), we get f^{-1}(cl(f(D))) is a J*P-closed set containing D. By the definition "The J*P-closure of D of a topological space (L,ρ) is defined as: J*Pcl(D)=∩{F⊆Y:D⊆F and F∈J*PC(Y,ρ)}", we have J*Pcl(D)⊆f^{-1}(cl(f(D))) which implies that f(J*Pcl(D))⊆cl(f(D)). Hence proved.

References:

1) Abd, M.E., Monsef, E.L., EI-Deeb, S.N. and Mohmoud, R.A. (1983), β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12,77-90.
2) Aslim, G., Caksu Guler,A . and Noiri,T.(2006) On πgs-closed sets in topological spaces,Acta. Math.Hungar.,112(4),275-283.
3) Baker,C.W.,(1996) On preserving g-closed sets,Kyungpook Math.J.,36,195-199
4) Balachandran,K.,Sundaram,P. And Maki,H.(1990) On generalized continuous maps in topological spaces,Mem.Fac.Sci.Kochi.Univ.Ser.A.Math.,12,5-13.
5) Caldas,M.,(1993) On g-closed sets and g-continuous Mappings,Kyungpook Math. J.,33(2),205-209.
6) Chawalit,B.,(2003a) Generalized continuous function from any topological space into product,Naresuan Univ.J.,11(2),93-98.
7) Dontchev, J (1996) Contra continuous functions and strongly S-closed spaces, International Journal of Mathematics and Mathematical Sciences, 19, 303-310.
8) Dunham, W., (1982) A New Closure Operator for Non-TI Topologies, Kyungpook Mathematical Journal, 22(1), 55-60.
9) Ekici,E. And Baker,C.W.(2007),On πg-closed sets and continuity,Kochi.J.of Mathematics,2,35-42.
10) Levine,N.(1960) Strong continuity in topological spaces,Amer.Math.Monthly,67,269-275.
11) Levine, N. (1963) Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70, 36-41.

12) Mashhour, A.S. et.al., (1982) On pre continuous and weak pre continuous functions, Proc. Math. Phys. Soc. Egypt, 53, 47-53.
13) Malini, R.(2022), J*P-Closed sets in topological spaces, Avinashilingam Institute for Home Science and Higher Education for women, Coimbatore.
14) Meenakshi, PL. and sivakamasundari. K., J-closed set in topological space, JETIR, Bol vol 6 (5) (2019), 193-201.
15) Meenakshi, PL. and sivakamasundari. K., J-open set in topological space, Research Highlights (2020), Avinashilingam Institute for Home Science and Higher Education for women, Coimbatore.
16) Meenakshi, PL. (2022), J-continuous functions in topological spaces , malaya Journal of Matematik, Vol. S, No. 1, 206-213, 2020.
17) Munshi, B.M. and Bassan, D.S.(1982) Super-Continuous Mappings, Indian J. Pure Appl.Math.,13(2),229-236.
18) Noiri,T.,(1980) On δ-continuous functions, J. Korean Math.Soc.,16,161-166**.**
19) Stone, M. (1937), Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41, 374-481.
20) Sundarm, P. and N. Nagaveni, (1998) On weakly generalized continuous maps, Far East J. Math. Sci., 6, 903-912.
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