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Abstract  View Article

## J-Closed Functions via J-Closed Sets in Topological Spaces

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In this article, J-closed functions using the concept of J-closed sets and J-open functions using the notion of J-open sets are initiated. The interrelationships of these newly introduced functions with various existing functions are examined and its properties are analysed. The composition of two J-closed functions need not be a J-closed function which is proved by counter example. The J-closed functions are used to define homeomorphisms using J-closed sets in topological spaces. Homeomorphisms are the isomorphisms in the category of topological spaces—that is, they are the mappings that preserve all the topological properties of a given space.

The study of J-closed functions has been done by the following methods:

- Analytical method of comparing J-closed functions with other existing closed functions.
- Obtaining counter examples wherever necessary to substantiate the result.
- Interpreting the results as diagrams.
- Analysis of preservation of topological properties by J-closed functions.





# J-Closed Functions via J-Closed Sets in Topological Spaces

P. L. Meenakshi<sup>a\*#</sup> and R. Sudha<sup>b#</sup>

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## Abstract

In this article, J-closed functions using the concept of J-closed sets and J-open functions using the notion of J-open sets are initiated. The interrelationships of these newly introduced functions with various existing functions are examined and its properties are analysed. The composition of two J-closed function need not be a J-closed function which is proved by Counter Example. The J-closed functions are used to define homeomorphisms using J-closed sets in topological spaces. Homeomorphisms are the isomorphisms in the category of topological spaces—that is, they are the mappings that preserve all the topological properties of a given space.

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- Analytical method of comparing J-closed functions with other existing closed functions.
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*Keywords:* J-closed function; J-closed set; J-open set; J-open function and  $\eta^*$ -open sets.

## 1 Introduction

In 1937, Stone [1] introduced regular open sets and used it to define the semi-regularization of a topological space. In 1968, Velicko [2] proposed  $\delta$ -open sets which are stronger than open sets. Levine [3] has brought generalized closed sets in 1970. Dunham [4] has established a generalized closure using Levine's generalized closed sets as  $Cl^*$ . In 2016, Annalakshmi [5] has introduced regular\*-open sets ( $r^*$ -open sets) using  $Cl^*$ . A class of new sets namely  $\eta^*$ -open sets, a union of  $r^*$ -open sets, which is placed between the classes of  $\delta$ -open set and open set is introduced. Its basic properties are procured and the concepts of  $\eta^*$ -cluster point,  $\eta^*$ -adherent point and a  $\eta^*$ -derived set are introduced and studied. Moreover, a new class of sets namely J-closed sets is initiated using  $\eta^*$ -open sets in topological spaces. This new concept is weaker than closedness and infact it is weaker than g-closedness but stronger than  $g\delta$ -closedness which are essential to characterize almost weakly Hausdorff spaces and thus the digital line. The properties and relationships with other g-closed sets are analysed. The class of J-closed sets is placed between that of generalized closed (g-closed) sets and generalized  $\delta$ -closed sets ( $g\delta$ -closed). Changing the position of  $\eta^*$ -Closure operator and  $\eta^*$ -openness another two concepts are introduced namely  $J^*$ -closed sets and  $J^{**}$ -closed sets and their status are analysed and properties are obtained. Many interesting characterizations are established. Finally in a semi-regular space,  $J, J^*$  and  $J^{**}$ -closed sets coincide which has to be proved by Example.

In this article, J-closed functions and J-open functions are initiated. The interrelationships of these newly introduced functions with various existing functions are examined and its properties are analysed.

## 2 Preliminaries

Notation 1.1 Through out the dissertation  $(Y, \zeta), (Z, \sigma)$  and  $(P, \mu)$  indicate topological spaces on which no separation axioms are referenced except as otherwise provided.

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